Bode Plot

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Bode Plot:- Bode plot is the frequency plot of sinusoidal Transfer function of a system. It consists of two plots. ① Magnitude plot (Magnitude of $G(j\omega)$ in dB Vs $\log \omega$) ② Phase plot (phase values of sinusoidal transfer function Vs $\log \omega$)

Basic factors of $G(j\omega)$

① constant factor, k

② Integral factor, $\frac{k}{s}$ (or) $\frac{k}{j\omega}$ $\quad \frac{k}{s^n}$ (or) $\frac{k}{(j\omega)^n}$

③ Differential factor, ks or $kj\omega$ $\quad ks^n$ (or) $k(j\omega)^n$

④ First order factor in Denominator, $\frac{1}{1+sT}$ (or) $\frac{1}{1+j\omega T}$ $\quad \frac{1}{(1+sT)^n}$ (or) $\frac{1}{(1+j\omega T)^n}$

⑤ First order function in numerator, $(1+sT)$ (or) $(1+j\omega T)$ $\quad (1+sT)^n$ (or) $(1+j\omega T)^n$

⑥ second order factors.

① **constant factor 'k'**:-

$G(s) = k$ and if $s = j\omega$; then $G(j\omega) = k$

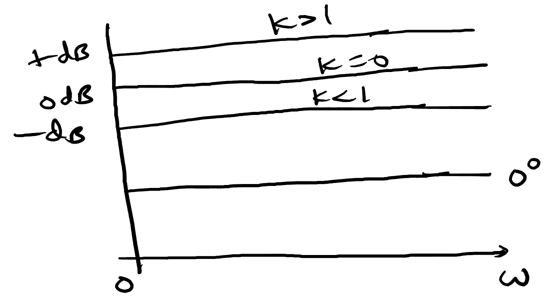
① Magnitude $|G(j\omega)| = |k| = k$

In dBs $= 20 \log k$.

① If $k = 1$; $20 \log 1 = 0$ dB

② If $k < 1$; $= -$dB

③ If $k > 1$; $= +$dB



② phase $\angle G(j\omega) = \angle k = \tan^{-1}\left(\frac{0}{k}\right) = 0°$

② **Integral factor**: $\frac{k}{s}$.

$G(s) = \frac{k}{s}$; $G(j\omega) = \frac{k}{j\omega}$

① Magnitude $|G(j\omega)| = \left|\frac{k}{j\omega}\right| = \left|\frac{-jk}{\omega}\right| = \frac{k}{\omega}$ $\qquad \left(|a+ib| = \sqrt{a^2+b^2}\right)$

(1) Magnitude $|G(j\omega)| = \left|\frac{k}{j\omega}\right| = \left|-\frac{jk}{\omega}\right| = \frac{k}{\omega}$ ($|a+ib| = \sqrt{a^2+b^2}$)

In dB's is $20 \log \frac{k}{\omega}$

Frequency 10 times (decade)

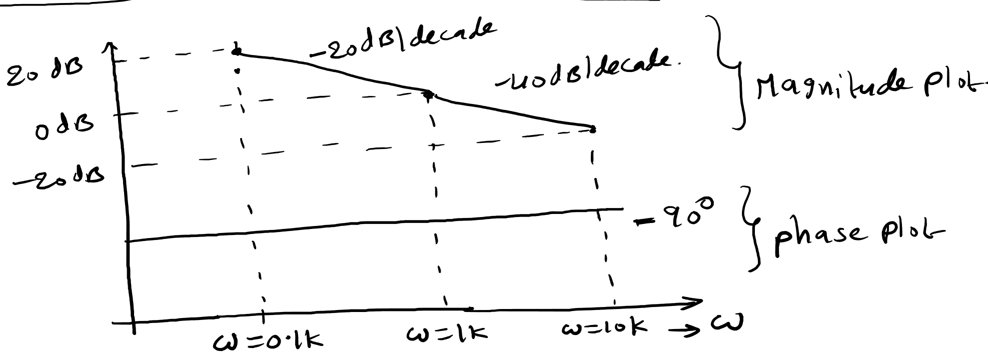
(1) If $\omega = 0.1k$; $20 \log \frac{k}{0.1k} = 20\,dB$
(2) If $\omega = k$; $20 \log \frac{k}{k} = 0\,dB$
(3) If $\omega = 10k$; $20 \log \frac{k}{10k} = -20\,dB$.

$-20\,dB/decade$

$\boxed{\frac{k}{s} \text{ factor causes the Gain to decrease by } 20\,dB \text{ for a frequency increment of 10 times}}$

(2) Phase. $\angle G(j\omega) = \angle \frac{k}{j\omega} = \frac{\angle k}{\angle j\omega} = \frac{\tan^{-1}\left(\frac{0}{k}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)} = \frac{0^\circ}{90^\circ} = 0^\circ - 90^\circ = -90^\circ$

$\boxed{\frac{k}{s} \text{ factor causes the phase of } -90^\circ}$



(3) Differential factor ks

$G(s) = ks$; $G(j\omega) = kj\omega$

(1) Magnitude $|G(j\omega)| = |kj\omega| = k\omega$

In dB's $20 \log |k\omega|$

Frequency 10 times

(1) If $\omega = \frac{1}{k}$ → $20 \log \left(k \cdot \frac{1}{k}\right) = 0\,dB$
(2) If $\omega = \frac{0.1}{k}$ → $20 \log \left(k \cdot \frac{0.1}{k}\right) = -20\,dB$
(3) If $\omega = \frac{10}{k}$ → $20 \log \left(k \cdot \frac{10}{k}\right) = 20\,dB$

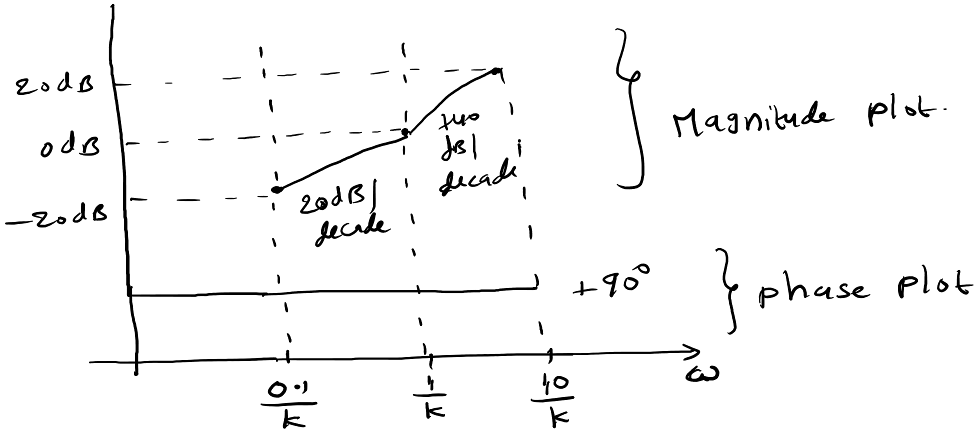
$-20\,dB/decade$

(2) phase:

$\angle G(j\omega) = \angle kj\omega = \angle k + \angle j\omega = \tan^{-1}\left(\frac{0}{k}\right) + \tan^{-1}\left(\frac{\omega}{0}\right) = 0^\circ + 90^\circ = 90^\circ.$

k factor increments by 20dB if frequency is increased 10 times

$\boxed{\text{ks factor increments by 20dB if frequency is increased 10 times and provides } +90^\circ \text{ phase shift.}}$



④ First order in denominator $G(s) = \frac{1}{1+sT}$; $G(j\omega) = \frac{1}{1+j\omega T}$

① Magnitude.

$$|G(j\omega)| = \left|\frac{1}{1+j\omega T}\right| = \frac{1}{|1+j\omega T|} = \frac{1}{\sqrt{1+\omega^2T^2}}$$

① At low frequencies i.e $\omega < \frac{1}{T}$; $1+\omega^2T^2 \approx 1$

$$\therefore |G(j\omega)|\Big|_{at\ \omega<\frac{1}{T}} \text{ is } \frac{1}{\sqrt{1}} = 1$$

In dB's $= 20\log 1 = \underline{0}$ dB.

② At high frequencies i.e $\omega > \frac{1}{T}$; $1+\omega^2T^2 \approx \omega^2T^2$

$$\therefore |G(j\omega)|\Big|_{at\ \omega>\frac{1}{T}} \text{ is } \frac{1}{\sqrt{\omega^2T^2}} = \frac{1}{\omega T}$$

In dB's $= 20\log\frac{1}{\omega T}$

$= -20\log\omega T.$

Frequency increase by 10 times

① If $\omega = \frac{1}{T} \rightarrow -20\log T\cdot\frac{1}{T} = 0$ dB

② If $\omega = \frac{10}{T} \rightarrow -20\log\frac{10}{T}\cdot T = -20$ dB.

Gain decrease by -20dB

② Phase:-

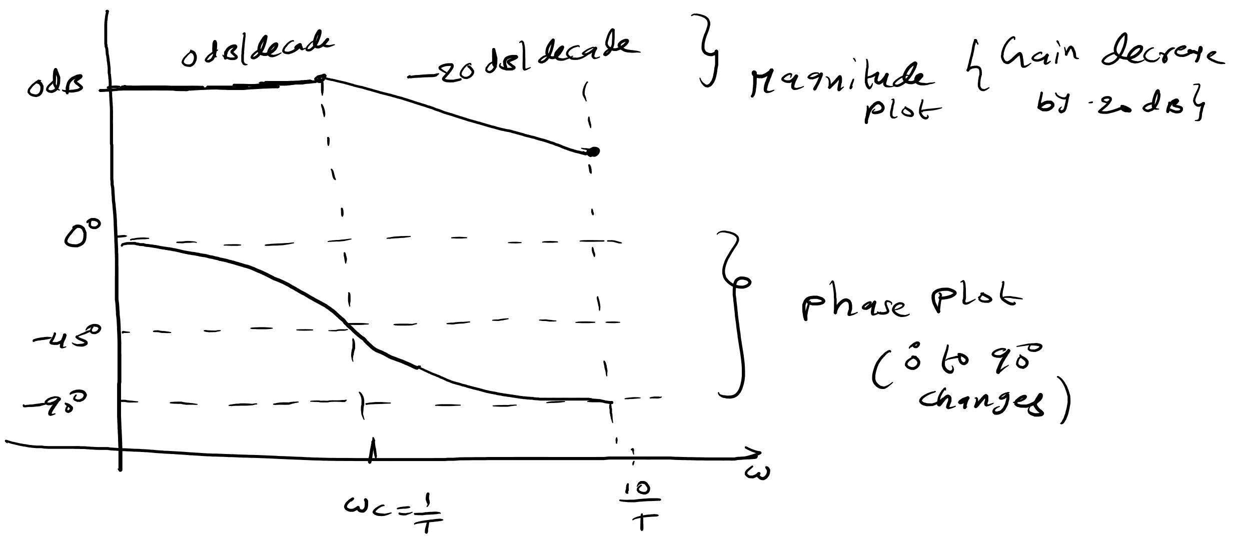
$$\angle G(j\omega) = \angle\frac{1}{1+j\omega T} = \frac{\angle 1}{\angle 1+j\omega T} = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{\omega T}{1}\right)} = \frac{0^\circ}{\tan^{-1}(\omega T)} = 0^\circ - \tan^{-1}(\omega T)$$

$$= \tan^{-1}(\omega T)$$

① $\omega = 0$; $\angle G(j\omega) = \tan^{-1}(\omega T) = \tan^{-1}(0) = 0^\circ$

② $\omega = \frac{1}{T}$; $\angle G(j\omega) = \tan^{-1}(\frac{1}{T} \cdot T) = \tan^{-1}(1) = 45^\circ$

③ $\omega = \infty$; $\angle G(j\omega) = \tan^{-1}(\infty) = \tan^{-1}(\infty) = 90^\circ$

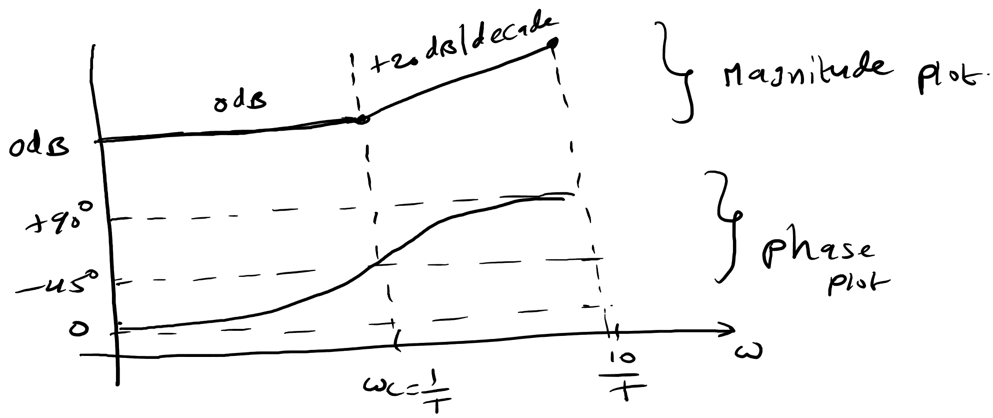


⑤ First order in numerator (1+sT)

solving same as above → we get.

① Magnitude increase by +20 dB| decade

② phase changes from 0° to -90°



point to conclude

		Magnitude	phase
		20 log K	0°
①	K		
②	$\frac{K}{s}$	-20 dB| decade	-90°
③	Ks	+20 dB| decade	+90°

{ $\frac{K}{s^n}$; n×-20dB   n×-90° }

{ Ks^n ; n×+20dB   n×+90 }

④ $\frac{1}{}$ { $\omega < \frac{1}{T}$; 0dB/decade → 0° to -90°
→ -45° at $\omega = \frac{1}{T}$ }

{ $\frac{1}{(1+sT)^n}$; n×-20dB   n×-90 }

④ $\frac{1}{1+sT}$ $\begin{cases} \omega < \frac{1}{T} ; \ 0\,dB/decade \\ \omega > \frac{1}{T} ; \ -20\,dB/decade \end{cases}$ → 0° to −90° → −45° at $\omega = \frac{1}{T}$ ← corner freq $\Big\}$ $\frac{1}{(1+sT)^n}$: $n \times -20dB$ $n \times -90$ }

⑤ $(1+sT)$ $\begin{cases} \omega < \frac{1}{T} ; \ 0\,dB/decade \\ \omega > \frac{1}{T} ; \ +20\,dB/decade \end{cases}$ → 0° to +90° → +45 at $\omega = \frac{1}{T}$ ← corner freq $\Big\{$ $(1+sT)^n$ $n \times +20dB$ $n \times +90$ }

⑥ second order { −40dB for Denominator −180° ; +40dB for Numerator +180° }

$$\left(1 + \frac{2j\omega\xi}{\omega n^2} + \frac{(j\omega)^2}{\omega n^2}\right)$$

corner freq $= \omega n$

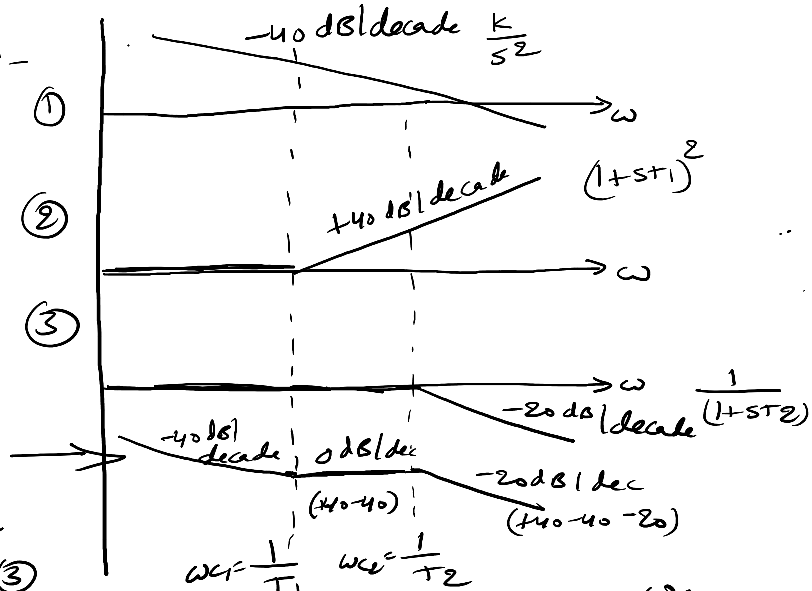
Corner Frequency

Examples corner frequency

① $(1+2s) \rightarrow$ corner freq $= \frac{1}{T} = \frac{1}{2}$ { i.e up to $\omega = \frac{1}{2}$; 0dB Magnitude ; $\omega > \frac{1}{2}$; +20 dB Magnitude }

② $\frac{1}{1+4s} \rightarrow$ corner freq $= \frac{1}{T} = \frac{1}{4}$ { i.e up to $\omega = \frac{1}{4}$; 0dB Magnitude ; $\omega > \frac{1}{4}$; −20 dB Magnitude }

Example :-



Final Resultant plot of ①②&③

Here $\omega c_1 < \omega c_2$

① $\frac{1}{T_1}, \frac{1}{T_2}$ are corner frequencies for terms $(1+sT_1)^2$ and $\frac{1}{(1+sT_2)}$

② Final plot up to ωc_1 i.e first corner frequency the final magnitude depends on $\frac{K}{s^2}$ term $(K, \frac{K}{s^n}, Ks^n)$

③ After first corner frequency i.e at ωc_2; the slope change is seen. Hence magnitude is calculated using formula.

Procedure for Drawing Bode plot

① Magnitude plot :-

① Convert the transfer function $G(s)$ into time constant form or Bode form.

$$K(1+sT_1) \longrightarrow G(j\omega) = \frac{K(1+j\omega T)}{j\omega(1+j\omega T_2)(\ldots(j\omega)^2 \ldots j\omega)}$$

or Bode form.

$$G(s) = \frac{k(1+sT_1)}{s(1+sT_2)\left(1+\frac{s^2}{\omega_n^2}+\frac{2\xi s}{\omega_n}\right)} \xrightarrow[s=j\omega]{} G(j\omega) = \frac{k(1+j\omega T)}{j\omega(1+j\omega T_2)\left(1+\frac{(j\omega)^2}{\omega_n^2}+\frac{2\xi j\omega}{\omega_n}\right)}$$

② List the corner frequencies in the increasing order and prepare table as follows.

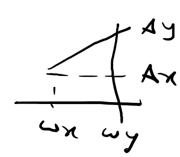
Term	Corner freq (rad/sec)	Slope (dB/decade)	Slope change (dB/decade)

In above table enter k or $\frac{k}{s^n}$ or ks^n terms first and the other terms must entered in increasing order of corner frequencies. Then enter corner frequency, slope contributed by each term and change in slope at every corner frequency.

③ choose arbitrary frequency ω_l; which is very much less than the lowest corner frequency.

calculate magnitudes of k, $\frac{k}{s^n}$, ks^n at ω_l and at the lowest corner frequency ω_{c1} using ($20\log_r$)

④ Then calculate the Gain in dB at other corner frequencies one by one using the formula.



$$\text{Gain at } \omega_y = \text{slope change from } \omega_x \text{ to } \omega_y \times \log\left(\frac{\omega_y}{\omega_x}\right) + \text{Gain at } \omega_x$$

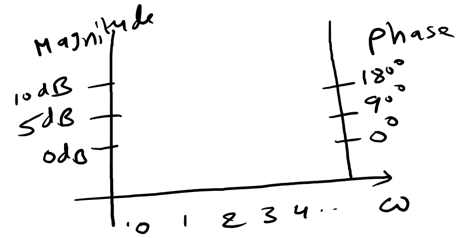
⑤ choose an arbitrary frequency ω_h; which is higher than the highest corner frequency and determine the magnitude at ω_h.

⑥ In semilog graph sheet mark the required range of frequency on x axis ($\log\omega$) and range of dB on Y-axis (normal scale).

⑦ Mark all the points and join the points using SCALE. Mark the slope at every point on graph

phase plot

phase values of $G(j\omega)$ is calculated at different values of 'ω' which are the 'ω' values considered for magnitude plot.



{ single Graph we Are Drawing Both Magnitude & phase plot }

problem 1:- Draw the Bode plot for following Transfer function.

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

Sol:- ① Convert Transfer function into time constant form (or) Bode form.

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)} \xrightarrow{s=j\omega} \frac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$$

Corner frequency $\omega_{c1} = \frac{1}{0.4} = 2.5$ $\left\{ \frac{1}{1+0.4s} \quad 1+sT_1 \quad \omega_{c1} = \frac{1}{T_1} \right\}$

$\omega_{c2} = \frac{1}{0.1} = 10$ $\left\{ \frac{1}{1+0.1s} \quad 1+sT_2 \quad \omega_{c2} = \frac{1}{T_2} \right\}$

② First fill $\frac{10}{s}$ term in following table.
Later fill with $\left(\frac{1}{1+0.4s}\right)$ term with corner frequency $\omega_{c1} = 2.5$
and then with $\left(\frac{1}{1+0.1s}\right)$ term with corner frequency $\omega_{c2} = 10$.
(i.e fill in order of Increasing corner frequencies)

Term	corner freq rad/sec	slope (dB/decade)	slope change (dB/decade)
$\frac{10}{j\omega}$	—	−20	—
$\frac{1}{1+0.4j\omega}$	$\omega_{c1} = 2.5$	−20	$-20-20=-40$
$\frac{1}{1+0.1j\omega}$	$\omega_{c2} = 10$	−20	$-40-20=-60$

$\frac{1}{1+0.1j\omega}$	$\omega_{c2} = 10$	-20	$-40-20=-60$

③ Choose $\omega_l \ll \omega_{c1}$.

Let $\omega_l = 0.1$ rad/sec.

④ Determine magnitudes at ω_l, ω_{c1}

① Magnitude at $\omega_l = 0.1$ rad/sec is $20\log\left|\frac{10}{j\omega}\right| = 20\log\left(\frac{10}{\omega}\right)$

$= 20\log\left(\frac{10}{0.1}\right) = 40$ dB

② Magnitude at $\omega_{c1} = 2.5$ rad/sec is $20\log\left|\frac{10}{j\omega}\right| = 20\log\left(\frac{10}{\omega}\right)$

$= 20\log\left(\frac{10}{2.5}\right) = 12$ dB

Gain at $\omega_{c2} = ?$

Gain at ω_{c2} = (change in slope from ω_{c1} to ω_{c2}) $\times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right)$ + Gain at ω_{c1}

$= (-40) \times \log\left(\frac{10}{2.5}\right) + 12$ dB

$= -12$ dB.

⑤ Determine $\omega_h \gg \omega_{c2}$

Let $\omega_h = 50$

Gain at ω_h = (change in slope from ω_{c2} to ω_h) $\times \log\left(\frac{\omega_h}{\omega_{c2}}\right)$ + Gain at ω_{c2}

$= (-60) \times \log\left(\frac{50}{10}\right) + (-12\text{dB})$

$= -54$ dB.

We got at

$\omega = \omega_l = 0.1 \rightarrow 40$ dB → a

$\omega = \omega_{c1} = 2.5 \rightarrow 12$ dB → b

$\omega = \omega_{c2} = 10 \rightarrow -12$ dB → c

$\omega = \omega_h = 50 \rightarrow -54$ dB → d

Phase plot:

$$\phi = \angle G(j\omega) = \angle \frac{10}{j\omega(1+j0.4\omega)(1+0.1j\omega)} = \frac{\tan^{-1}\left(\frac{0}{10}\right)}{\tan^{-1}(\omega)\ \tan^{-1}(0.4\omega)\ \tan^{-1}\left(\frac{0.1\omega}{1}\right)}$$

$$\phi = \angle G(j\omega) = \frac{\angle 10}{\angle j\omega \ \angle 1+0.4j\omega \ \angle 1+0.1j\omega} = \frac{\tan^{-1}\left(\frac{0}{10}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right) \ \tan^{-1}\left(\frac{0.4\omega}{1}\right) \ \tan^{-1}\left(\frac{0.1\omega}{1}\right)}$$

$$\frac{\angle A}{\angle B} = \angle A - \angle B$$

$$\angle A \cdot \angle B = \angle A + \angle B$$

$$= \frac{0^\circ}{90^\circ + \tan^{-1}\left(\frac{0.4\omega}{1}\right) + \tan^{-1}\left(\frac{0.1\omega}{1}\right)}$$

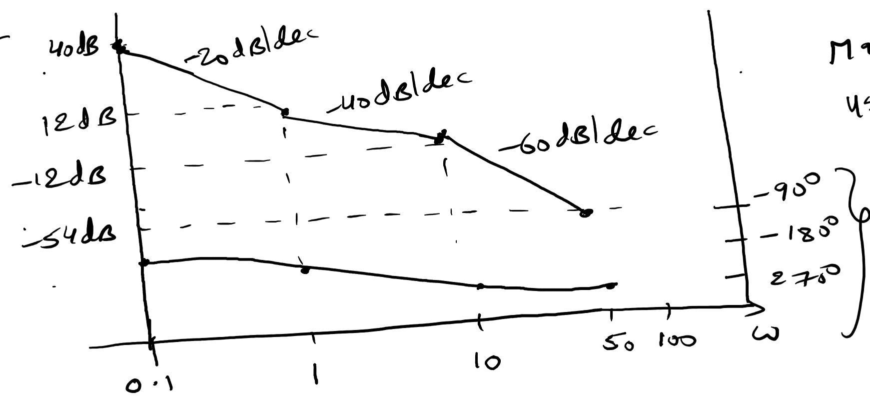
$$= 0^\circ - 90^\circ - \tan^{-1}\left(\frac{0.4\omega}{1}\right) - \tan^{-1}\left(\frac{0.1\omega}{1}\right)$$

$$\therefore \boxed{\angle\phi = -90^\circ - \tan^{-1}(0.4\omega) - \tan^{-1}(0.1\omega)}$$

ω	$\tan^{-1}(0.4\omega)$	$\tan^{-1}(0.1\omega)$	$-90^\circ - \tan^{-1}(0.4\omega) - \tan^{-1}(0.1\omega)$	
0.1	2.29°	0.57°	-92°	→ e
1	21.8°	5.7°	-116°	→ f
2.5	45°	14.03°	-150°	→ g
4	57.9°	21.80°	-170°	→ h
10	75.96	45°	-210°	→ i
50	87.13	78.69	-256°	→ j

Final Plot

Magnitude



phase

Magnitude plot using scale (straight line)

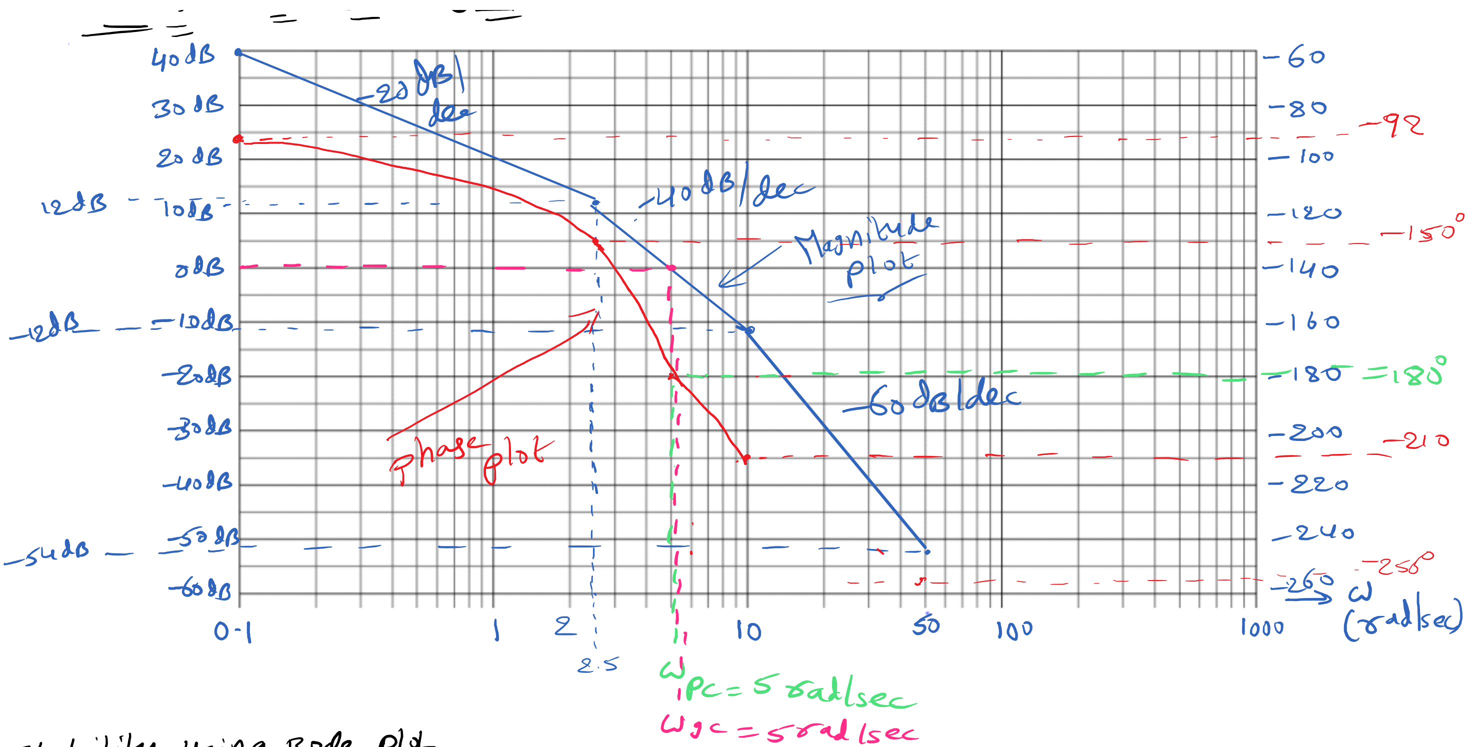
phase plot

smooth curve with HAND

Graph :- Take 1 unit = 5 dB on y-axis on left hand side and
1 unit = 10° on x-axis on Right hand side.

on x-axis start frequency $\omega = 0.1$ rad/sec

Bode plot on semilog graph



stability using Bode plot

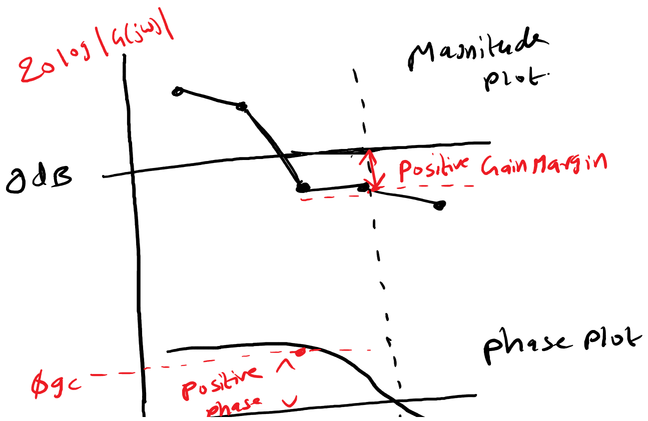
1. Gain Margin & phase Margin is positive {stable} $\omega_{gc} < \omega_{pc}$
2. Gain Margin & phase Margin is negative {unstable} $\omega_{gc} > \omega_{pc}$
3. Gain Margin = 0 & phase Margin = 0 {Marginally stable} $\omega_{gc} = \omega_{pc}$

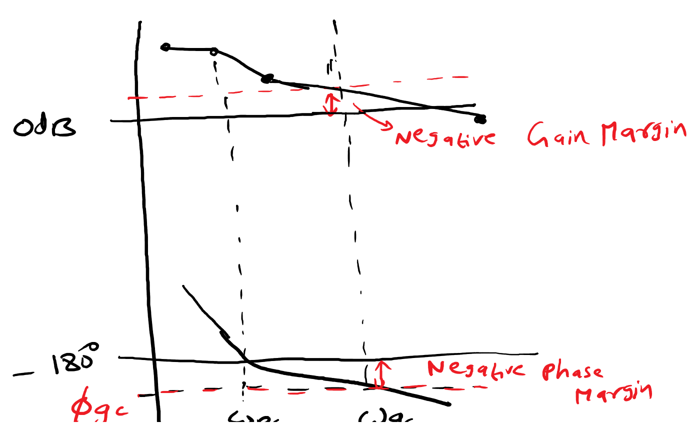
(1) Gain cross over frequency (ω_{gc}) :- Frequency at which gain is unity. i.e frequency at which magnitude plot crosses 0 dB.

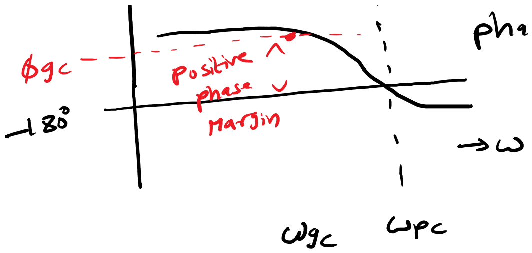
(2) Phase cross over frequency (ω_{pc}):- Frequency at which phase crosses $-180°$.

(3) Gain Margin (k_g):- Reciprocal of magnitude of open loop T.F at phase cross over frequency. $k_g = \frac{1}{|G(j\omega)|}\Big|_{\omega=\omega_{pc}} \Rightarrow -20\log|G(j\omega)|\,\big|_{\omega=\omega_{pc}} \Rightarrow -20\log|G(j\omega_{pc})|$

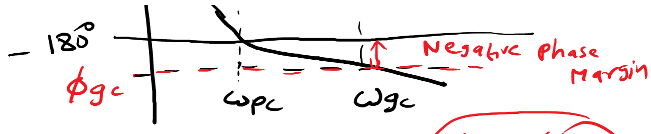
(4) Phase Margin (γ):- phase to be added at gain crossover frequency to bring system to verge of instability. $\gamma = \phi_{gc} + 180°$







$\omega_{gc} < \omega_{pc}$ stable



$\omega_{gc} > \omega_{pc}$ unstable

(1) Gain Margin $= -20\log|G(j\omega)|\big|_{\omega=\omega_{pc}}$

(2) phase Margin $= \phi_{gc} + 180°$

$= \text{phase at } \omega_{gc} + 180°$

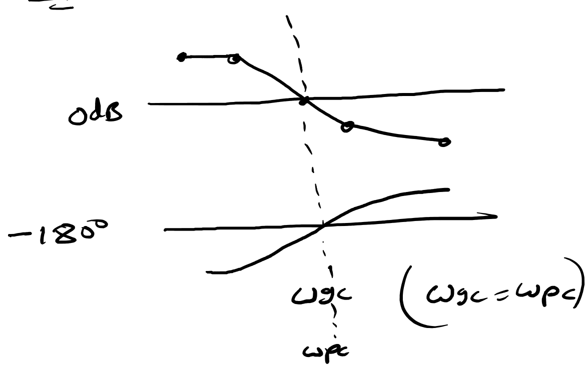
Gain Margin, phase Margin

+ve → stable

−ve → unstable

= 0 → Marginally stable

Marginally stable



Zero phase Margin

Zero Gain Margin

Problem II :- Sketch the Bode plot for following Transfer function & Determine gain cross over frequency, Phase cross over frequency, Gain Margin and Phase margin.

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

Sol:- Step 1:- Convert the given Transfer function in to Time Constant form or bode form.

$$G(s) = \frac{75(1+0.2s)}{100 \cdot s\left(\frac{s^2}{100} + \frac{16s}{100} + 1\right)} = \frac{0.75(1+0.2s)}{s(0.01s^2 + 0.16s + 1)}$$

Substitute $s = j\omega$

$$G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega(0.01(j\omega)^2 + 0.16j\omega + 1)}$$

$$= \frac{0.75(1+0.2j\omega)}{\ldots}$$

$$= \frac{0.75(1+0.2j\omega)}{j\omega(1-0.01\omega^2+0.16j\omega)}$$

Terms are (1) $\frac{0.75}{j\omega}$, (2) $1+0.2j\omega$

$\omega_{c1} = \frac{1}{0.2} = 5$ rad/sec

(3) $\frac{1}{(1-0.01\omega^2+0.16j\omega)}$

$\omega_{c2} = \omega_n = 10$ rad/sec

(2) Fill the following table by writing $k, ks^n, \frac{k}{s^n}$ terms first and then increasing order of corner frequencies.

Term	Corner freq rad/sec	dB/dec slope	dB/dec slope change
			–
$\frac{0.75}{j\omega}$	–	−20	−20 + 20 = 0
$(1+0.2j\omega)$	5 (ω_{c1})	+20	
$\frac{1}{j\omega(1-0.01\omega^2+0.16j\omega)}$	10 (ω_{c2})	−40	0 − 40 = −40

(3) Choose ω_l; which is much less than lower corner frequency ω_{c1}

$\omega_l \ll \omega_{c1}$ $\omega_l \ll 5$

so choose $\omega_l = 0.5$.

(4) Determine magnitudes at ω_l, ω_{c1}.

(1) Magnitude at $\omega_l = 0.5$ is $20 \log \left|\frac{0.75}{j\omega}\right| = 20 \log \frac{0.75}{\omega}$

$= 20 \log \frac{0.75}{0.5} \cong 3.5$ dB

(2) Magnitude at $\omega_{c1} = 5$ is $20 \log \frac{0.75}{5} = -16.5$ dB

(3) Magnitude at $\omega_{c2} = 10$ is ?

③ Magnitude at $\omega_{c2} = 10$ is ?

Gain at ω_{c2} = Gain at ω_{c1} + change in slope from ω_{c1} to ω_{c2} $\times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right)$

$$= -16.5 + (0\text{ dB}) \times \log\left(\frac{10}{5}\right) = -16.5\text{ dB}$$

⑤ choose ω_h which is greater than ω_{c2}.

$$\omega_h > \omega_{c2} \qquad \omega_h > 10$$

choose $\omega_h = 50$.

Gain at ω_h = Gain at ω_{c2} + change in slope from ω_{c2} to ω_h $\times \log\left(\frac{\omega_h}{\omega_{c2}}\right)$

$$= -16.5 + (-40)\log\left(\frac{50}{10}\right)$$

$$= -44\text{ dB}.$$

As $\omega\uparrow$ Gain Decreasing { $\omega \to \infty$ then amplitude is tends to $-\infty$ }

<u>Phase</u>

$$\angle G(j\omega) = \angle \frac{0.75(1+0.25j\omega)}{j\omega(1-0.01\omega^2+0.16\omega j)}$$

$$\frac{\angle 0.75 \ \angle 1+0.25j\omega}{\angle j\omega \ \angle (1-0.01\omega^2+0.1\omega j)} = \frac{\tan^{-1}\left(\frac{0}{0.75}\right) + \tan^{-1}(0.25\omega)}{\underbrace{\tan^{-1}\left(\frac{\omega}{0}\right)}_{90^\circ} + \tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)}$$

$$= 0^\circ + \tan^{-1}(0.25\omega) - 90^\circ - \tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$$

$$\angle\phi = -90^\circ + \tan^{-1}(0.25\omega) - \tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$$

For second order terms the angle varies from 0° to 180°. But $\tan^{-1}$ gives upto 90. so Add 180° for $\omega > \omega_n$

$$\angle\phi = -90 + \tan^{-1}(0.25\omega) - \tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right) \qquad \omega < \omega_n$$

$$= -90 + \tan^{-1}(0.25\omega) - \left[\tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right) + 180^\circ\right] \qquad \omega > \omega_n$$

<u>Phase Plot</u> :-

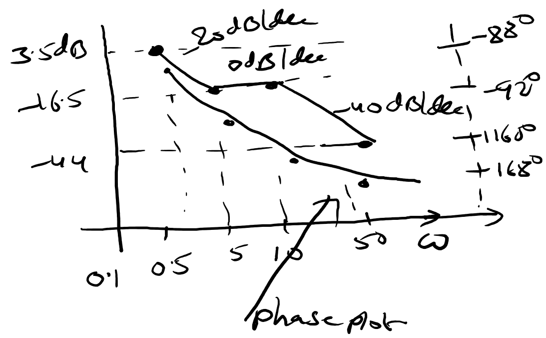
Phase plot :-

ω	$\tan^{-1}(0.2\omega)$	$\tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$	$\angle\phi$	
0.5	5.71°	4.6	-88°	e
1	11.3°	9.2	-88°	f
$\omega_{c1}=5$	45°	46.2	-92°	g
$\omega_{c2}=10$	63.43°	90	-116°	h
20	75.96°	$-46.8+180=133.2$	-148°	i
$\omega_h=50$	84.28°	$-18.4+180=161.6$	-168°	j
100	87.13°	$-92+180=170.8$	-174°	k
∞			-180°	l

$> \omega_n$

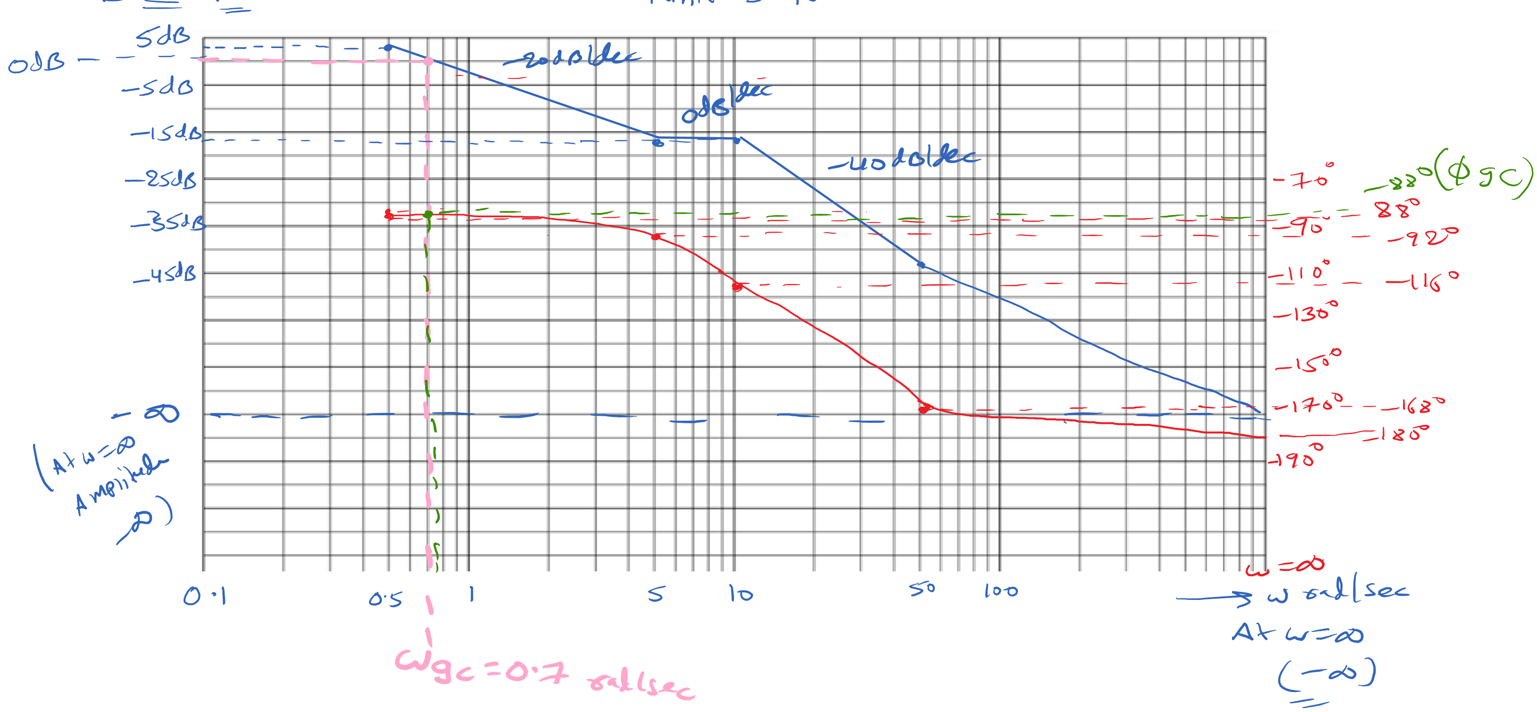
Now

	Magnitude	phase
$\omega_e = 0.5$	3.5 dB	-88°
$\omega_{c1} = 5$	-16.5 dB	-92°
$\omega_{c2} = 10$	-16.5 dB	-116°
$\omega_h = 50$	-44 dB	-168°



Bode plot:-

on y axis 1 unit = 5dB
1 unit = 10°



① Phase Margin $\gamma = 180° + \phi_{gc}$

+ (Angle at ω_{gc}).

$= 180^\circ - 88^\circ$

$= 92^\circ$ (+ve)

② Gain Margin $\quad K = -(20 \log |G(j\omega)|) \Big|_{at\ \omega = \omega_{pc}}$

Here $\omega_{pc} = \infty$.

$= -($Magnitude in dB at $\omega = \omega_{pc})$

ω_{pc} at ∞ then Amplitude is $-\infty$

$= -(-\infty)$

$= +\infty$ (+ve)

Gain Margin & phase margin are positive

so system is stable.